GovReg
WORKING PAPER
SERIES

Uninformed Bidding in Sequential Auctions

Emmanuel Lorenzon

December 9, 2020

RESEARCH
Dauphine | PSL
CHAIRE GOUVERNANCE
ET RÉGULATION

Uninformed Bidding in Sequential Auctions*

Emmanuel Lorenzon[1,2]

[1]University Grenoble Alpes, CNRS, INRAE, Grenoble INP, GAEL, 38000 Grenoble, France
[2]Governance and Regulation Chair, University Paris-Dauphine, PSL Research University, F-75016 Paris, France

December 9, 2020

Abstract

We consider a private value auction with one-sided incomplete information in which two objects are sold in a sequence of two second-price auctions. Buyers have multi-unit demands and both are asymmetrically informed at the *ex-ante* stage of the game. One buyer perfectly knows his type and the other is uninformed about his own type. We consider *interim* information acquisition for the *uninformed* buyer and derive an asymmetric equilibrium which is shown to produce a declining price sequence across both sales. The supermartingale property of the price sequence stems from the *uninformed* buyer's incentives to gather private information which leads to an aggressive bidding at the first-stage auction.

Keywords: Sequential auctions ; Uninformed bidding ; Multi-unit demand ; Declining price anomaly

JEL Classification: D44; D82; L86; M37

*Acknowledgements: I am grateful to Petros Sekeris, Wilfried Sand-Zantman, David Ettinger, Noemi Navaro, Nicolas Carayol, David Malueg and Laurent Bergé for their helpful comments and suggestions. All errors are mine.

1 Introduction

Individual buyers may not be fully aware of how much they would pay when deciding which object to acquire, either because they do not need to fully assess a value or simply because they do not have access to sufficient information. Consider an auction of several identical units put on sale sequentially involving an expert and a non-expert buyer. Both of them seek several units and the non-expert buyer learns the value of each unit only after experimenting one. At each round, the buyers must offer a price simultaneously. How should the buyers behave, and what will the equilibrium path price be throughout the sales?

We consider the sale of online display advertising, a market in which a publisher serves as a platform to bring together consumers and firms. Each platform captures its users attention and resells it to a pool of firms seeking to advertise their products on the platform's website for a typical subset of its users (see Prat and Valletti (2018)), i.e., through targeted advertising. Consider two platforms selling an homogenous targeted ad slot, one after another, using a second-price auction (where one corresponds to one consumer) at an AdExchange market place (see McAfee (2011)). Each time a user loads a publisher's web page, a second-price auction is implemented for the right to display one's own advertisement. Advertisers bid for an impression—the impression of the advertisement on the webpage—and have access to information on both the publisher and the user's characteristics (gender, preferences, past clicks, location, and so on.).

Suppose that a buyer, here a producer selling a good to consumers, is already established in the market and knows consumers behaviour perfectly, and consider another buyer, which is a new entrant, and may discover the value of the ad only after a user clicks on it. As pointed out by Lewis and Rao (2015), the monetary returns of an impression depends on the buyer's ability to gather information about customers' behavioural patterns and to build a knowledge package of past clicks and buys. Hence, advertisers with high capabilities construct accurate bids on the basis of consistent proprietary information and a proper estimation of profit opportunities. Yet many advertisers enter the auction game unaware of such opportunities. From an interim perspective, former advertisers have a strategic advantage over newcomers, who only collect relevant information by "experimenting" the impression. Consequently, a poorly-*informed* advertiser might only consider the risk of over-estimating the value of an impression.

We characterize equilibrium bids assuming private valuations and two asymmetrically informed buyers asking for more than one unit[1]. Buyers are asymmetrically informed about their types at the *ex-ante* stage. One buyer learns of his type *ex-ante* the first auction, whereas the other has no *prior* information, except the distribution from which types are drawn. We consider a costless *interim* information acquisition, which requires that each unit is allocated and experienced at the end of each auction. The *uninformed* buyer acquire information about his type at the end of the first auction if he wins the first unit. We derive the equilibrium from such a game and study thereafter the resulting effect on the equilibrium price sequence.

[1]In such a setting, Celis et al. (2011) show that bidders' valuations are uncorrelated and private within AdExchange auctions, so that assuming an independent private-value environment is a reasonable assumption. Furthermore, each ad is sold for a specific type of consumer that firms typically target when buying an ad slot, so that considering a stable set of advertisers in each auction is also a reasonable assumption.

Related work One-sided incomplete information in sequential auctions has mostly been considered with single-unit demands under the common values paradigm (see Engelbrecht-Wiggans and Weber (1983), Hausch (1986), Bikhchandani (1988), and Hörner and Jamison (2008)) or under a first-price structure when buyers have discrete private valuations (see Jeitschko (1998), Kannan (2010)). Few studies analyse informational issues with multi-unit demand, and mainly focus on the first-price rule in which the effect of information transmission through bids is strong (see Février (2003) and Ding et al. (2010)). We study informational asymmetries in a sequential second-price auction with multi-unit demand and private values.

With multi-unit demand, bids are driven by an indifference condition between winning a unit now and losing the other one tomorrow; and losing the first now and winning the other unit tomorrow. From this trade off, a symmetric first-round equilibrium strategy calls for a buyer to bid the price expected to be paid at the second auction provided he has the highest valuation among his competitor[2]. With *ex-ante* one-sided incomplete information, asymmetries give the *informed* player an advantage as he can either keep his rival uncertain or allow for a learning experience and take advantage of it. In our model, this player accounts for the opportunity *gain* to let his competitor acquire information and trades off the costs and benefits from a *prevent-to-learn* strategy at the first auction. The *uninformed* player's action is driven by the opportunity to learn private information (the first unit bears both a consumption and an informational value) and the player trades off the costs and benefits from becoming informed. The *informed* player underbids with respect to his valuation, whereas the *uninformed* player bids aggressively and the model results in an asymmetric equilibrium.

Milgrom and Weber (2000) and Weber (1983) show that with symmetrically informed buyers and constant valuations, winning prices should follow a martingale or drift upward. Yet, since Ashenfelter (1989) and Beggs and Graddy (1997), many of empirical studies find evidence of a decline in price at sequential sales, referred to as *the declining price anomaly* or *the afternoon effect*[3]. From this documentation, a major focus has been to study the price properties. For instance, McAfee and Vincent (1993) show that the price decline may stem from buyers' risk aversion. von der Fehr (1994) explains it by the existence of participation costs. Bernhardt and Scoones (1994) and Engelbrecht-Wiggans (1994) find that prices decline if buyers have stochastic demands, whereas for Jeitschko (1999) the decline stems from supply uncertainties. Considering a broader class of mechanisms including different price announcement policies, Kittsteiner et al. (2004) find that buyers' impatience causes the declining price path both under first and second-price rules. When buyers ask for more than one unit, Black and de Meza (1992) explain the decline in prices by the existence of an option to buy, while Jeitschko and Wolfstetter (2002) and Menezes and Monteiro (2003) show that the decline persists if buyers experience positive synergies when winning several units.

In these studies, buyers are symmetrically informed at each round and there is a short interval of time between sales—except for in the paper by Kittsteiner et al. (2004) who include time preferences that postpones private learnings.

To translate our framework to standard arguments, we consider an aggregated impression—

[2] See, for instance, Black and de Meza (1992), Jeitschko and Wolfstetter (2002), and Menezes and Monteiro (2003).

[3] Though, increasing patterns have been highlighted by Gandal (1997), Raviv (2006) or Deltas and Kosmopoulou (2004).

as a proxy—auctioned at different points in time. In this perspective, our model is representative of the sale of two identical units in an interval of time, a period that is not too long for discounts but long enough to learn the value of a unit allocated immediately at the end of a sale[4].

We find that conditional on the first price, the expected second price is below the first one and prices drift downward on the basis of the incentive to acquire information from the *uninformed* buyer. This pattern is strengthened if the same buyer wins both units and this buyer is informed. However, our result does not rule out the possibility of an *ex-ante* increasing trend. It is observed that if the *uninformed* buyer wins both units then the expected second price should be higher than the first, which makes sense with the multi-unit demand assumption as the *informed* buyer shades his first-round bid in equilibrium.

2 A model with asymmetrically-informed buyers

A seller owns two identical objects for which he has zero reserve price and puts them on sale through a sequence of two second-price auctions. There are two potential buyers (bidders). Each buyer i's valuation for one object is denoted by x_i and is drawn from a common knowledge distribution function G with $supp\,(G) = [0, \bar{x}]$, $G\,(0) = 0$ and positive density g everywhere on its support. We consider an independent private value setting, that is, buyer i's valuation is private information and is independent from his competitor's valuation. Buyer i has multi-unit demand; that is, each buyer desires both objects. We consider that private valuations for each unit are perfectly correlated, that is, $X_i^1 = X_i^2$, so that exposure effects are postponed, and the game has no synergies between auctions. Finally, we denote by Ex_i the expected value of x_i.

The setup differs from the standard private value setting in that we consider both buyers to be asymmetrically informed. One buyer, denoted U, is uninformed about his valuation X_U *ex-ante* the first auction, whereas the other one, buyer I, does learn his own valuation X_I *prior* to when the first auction starts. Neither of them knows the valuation of his competitor, only that it is drawn from distribution G. It is considered that once the uniformed buyer wins a unit, he perfectly and costlessly learns the realized value of X_U.

In each round, both players simultaneously submit a bid b_i^t with $i = \{I, U\}\,, t = \{1, 2\}$. Let the function $\beta_i : [0, \bar{x}] \mapsto \mathbb{R}_+$ to be a *pure strategy* for buyer $i = \{I, U\}$ that assigns a bid $b_i^t = \beta_i^t(x_i)$ to each type $x_i \in [0, \bar{x}]$ in period t. The winner in each stage earns a payoff equal to his valuation discounted by the bid of his competitor and the runner-up earns nothing. Players are risk neutral and buyer i's preference in period t is represented by the quasi-linear expected utility payoff function π_i^t, which given the opponent's strategy b_j^t equals:

$$\pi_i^t\,(\mathbf{x}) = E\left[\left(x_i - b_j^t\right)\mathbb{1}\left\{b_i^t > b_j^t\right\}\right] \quad i, j = U, I,\ i \neq j \quad \textit{and} \quad t = 1, 2$$

The game proceeds as follows: (1) *Prior* to the first auction, player I learns his valuation X_I whereas player U does not and only knows that both are drawn from G; (2) A first auction is held in which both players announce simultaneously a bid as a function of their private

[4]Related auctions would be the sequential sales of used cars, fish auctions or rare book auctions (in which a library waits some time to gather a sufficient stock of books to be auctioned).

information, the highest buyer wins the first unit; (3) The seller allocates the unit to the winner of this auction stage, if player U is the winner then he learns perfectly the realized value of X_U; (4) A second auction is held in which again both players announce a bid simultaneously; (5) The seller allocates the last unit to the winner of this stage.

The game is solved backward, and consequently the equilibrium concept used is the non-cooperative perfect Bayesian equilibrium.

3 The equilibrium and price trend

The second auction stage is a one-shot Vickrey auction in which standard arguments apply. During the second auction, buyers have a dominant strategy, irrespective of what happened before so that it allows us to focus on the first-period equilibrium bids and to drop out issues related to belief updates after observing any realized information at the end of the first stage.

3.1 Equilibrium bids

We begin this section by recalling that if both players were to be symmetrically informed and both have constant valuations for the two units, then bidding truthfully in both auctions is a dominant strategy.

Symmetric equilibrium: In a sequence of two second-price auctions, a symmetric equilibrium involves buyers bidding in the first round the price they expect to pay at the second auction. In equilibrium, the optimal bid function of buyer i is derived from an indifference condition toward his rival having a valuation close enough to his valuation. Hence, if his rival has the same valuation, then the two units are split between both buyers. Therefore, buyer i should be indifferent between winning the first unit and losing the second one; and losing the first unit and winning the second one provided that the runner-up is of the same type. Assuming that players use a symmetric increasing bidding strategy $\beta_i^1(x_i)$ and denote by Y_1 the valuation of his rival, one obtains the following relation:

$$x_i - \beta_i^1(x_i) + 0 = 0 + x_i - E[Y_1|Y_1 = x_i],$$

and the first period auction results in a symmetric equilibrium bid function of $\beta_i^1(x_i) = x_i$. Both prices (and revenues) are determined by the runner-up's valuation and remain constant between the two auctions just like under the single-unit demand assumption (Milgrom and Weber (2000), Weber (1983)). When one player becomes strictly *uninformed* about his own type, things are different. Players are no longer symmetric at the first-stage auction, and the expectation by buyer i of his rival's type only matters if a learning experience takes place during the course of the game.

Player I's equilibrium strategy: The asymmetry creates an advantage for the *informed* player. His rival must decide a first-round bid according to the inference he is able to make about his type, which is driven by the probability function used by the Nature. The only inference he has is Ex_U, which by assumption of *iid* valuations is common knowledge. The *informed* player can therefore keep his rival uncertain to secure at least one unit with probability one if he is of a type lower than Ex_U. Consequently, the indifference condition is made on the basis of this trade-off and players act asymmetrically at the first stage.

Consider that the two units are sold by a sequence of two English auctions. The *informed* player should be indifferent between winning the first unit at price $\tilde{p}$ and the second unit, which prevents information acquisition; and losing the first unit and winning the second, which allows for a learning. One obtains the following relation:

$$x_I - \tilde{p} + (x_I - Ex_U)^+ = 0 + E\left[(x_I - Y)_{\mathbb{1}\{x_I > Y\}}\right]$$

in which $(f(x))^+$ means $\max\{0, f(x)\}$ and stems from individual rationality at each stage, that is, the utility payoff is at least null at each auction. The optimal price at which the *informed* player should drop out is thus $\tilde{p} = x_I - \left[\pi_I^2(.|lost) - \pi_I^2(.|won)\right]$. It is optimal for him to bid his valuation discounted by the difference in utility payoff from winning the second unit if he loses the first one or if he wins it.

Let $\sigma_I = \left(b_I^1, b_I^2\right)$ to be the strategy profile of the *informed* buyer composed by the first period bid function $b_I^1 = \tilde{p}$ and the second auction bid function $b_I^2 = x_I$. The next result shows that this rationale carries on as a Bayes-Nash equilibrium strategy for the sealed-bid format considered in this note and that indeed it is optimal for the *informed* buyer to act conservatively in the first auction:

Lemma 1. *$\sigma_I = \left(b_I^1, b_I^2\right)$ forms an equilibrium strategy profile in which the* informed *player underbids at the first auction and bids truthfully at the second. Formally, $\sigma_I = \left(b_I^1, b_I^2\right)$ is defined by:*

$$\sigma_I = \begin{cases} b_I^1 = x_I\left(1 - G\left(x_I\right)\right) + \left(x_I - Ex_U\right)^+ + E\left[X_U | X_U \leq x_I\right] G\left(x_I\right) \\ b_I^2 = x_I \end{cases}$$

Proof. See appendix A.1. □

Along the equilibrium path, player I bids more conservatively at the first auction than what he would have played in a one-shot Vickrey auction, and he bids truthfully in the second auction. In determining his equilibrium strategy for the first auction, the *informed* player accounts for the opportunity cost related to using a *prevent-to-learn* strategy, and the shading in lemma 1 reflects his incentives to let his opponent acquire information. He bids his valuation discounted by the opportunity cost of being runner-up during the first auction. To see this, we straightforwardly write the equilibrium first period bid as follows:

$$b_I^1 = x_I + (x_I - Ex_U)^+ - \int_0^{x_I} (x_I - x_U)\, dG(x_U) = x_I + \left(\Pi_I^2(.|win) - \Pi_I^2(.|lost)\right),$$

in which Π_I^2 and Π_I^2 denote, respectively, the second period payoff after winning and losing the first unit for player I. Since in equilibrium $b_I^1 \leq x_I$ we have that $x_I + \left(\Pi_I^2(.|win) - \Pi_I^2(.|lost)\right) \leq x_I \Leftrightarrow \Pi_I^2(.|win) \leq \Pi_I^2(.|lost)$. Hence, it is profitable for player I to lose the first unit.

For bidder I's types greater than Ex_U, information acquisition is consequential only if in the second stage player U turns out to have a stronger type. While winning the first auction secures the second unit, letting bidder U acquire information is profitable if bidder U obtains a type lower than Ex_U. Hence, bidder I accounts for the expected loss he would incur in the second auction following a learning experience and we have that $b_I^1 = x_I - (E[X_U|X_U > x_I] - x_I)(1 - G(x_I))$.

Conversely, for bidder I's types lower than Ex_U, winning the first auction entails a future payoff of zero. In this case, letting his competitor acquire information yields a positive payoff at the second auction, while keeping a positive expected payoff at the first auction. As a result, Bidder I trades-off between two events: he can win the first unit, keeping bidder U uninformed, which yields a strict loss in the second auction, or he can lose the first unit, which yields a positive gain with positive probability in the second-period auction. More precisely, player I accounts for the expected gain he would obtain in the second auction if his competitor turns out to have a lower type and, we have that $b_I^1 = x_I - (x_I - E[X_U|X_U < x_I])\, G(x_I)$.

Player U's best response: Let us now consider the optimal first-period decision of the *uninformed* player.

Property 1. *We denote by $\gamma = \left[\beta_I^1\right]^{-1}$ and $\phi = \left[\beta_I^1\right]^{-1}$ respectively the inverse functions of $\beta_I^1(.)$ for $x_I \in [0, Ex_U]$ and $x_I \in [Ex_U, \bar{x}]$. Functions ϕ and γ are both convex and such that $\phi(x) \geq x$ and $\gamma(x) \geq x$.*

This property comes from the fact that the piece-wise function β_I^1 is concave on both $[0, Ex_U]$ and $[Ex_U, \bar{x}]$ and such that $\beta_I^1(x_I) \leq x_I$ and will be used throughout the computation of player U's expected payoff.

As for player I, a similar indifference condition can be drawn for player U's decision. However, the *uninformed* player accounts for the possibility of being informed at the beginning of the second auction, and he trades-off the costs and benefits from the information acquisition at the end of the first auction. The *uninformed* player should be indifferent between being informed at the end of the first stage by paying a price b_0 (which would be his cost of perfect information acquisition), winning the second unit, and staying in the "dark" for the second auction. We obtain the following relation,

$$Ex_U - b_0 + \mu H(b_0) = 0,$$

in which we denote by $H(b_0) \geq 0$ player U's expected profits from learning at price b_0 at the end of the first auction and μ the probability of winning the first stage. Notice that if he stays uninformed for the second auction game (when he lost the first auction), then, *ex-ante*, his expected payoff from participating at this stage should be null, since in expectation both players have the same value. It follows that the optimal price b_0^* at which the *uninformed* player should drop out is solution of $b_0 - Ex_U = \mu H(b_0)$ and is at least equal to Ex_U.

Let $\sigma_U = \left(b_U^1, b_U^2\right)$ be the strategy profile of the *uninformed* bidder composed by the first period bid function $b_U^1 = b_0^*$ and the second auction bid function $b_U^2 = \{Ex_U, x_U\}$. The next result characterizes the optimal first period behaviour of the *uninformed* player and shows that in equilibrium the incentives to gather information drive up his bid to a level higher than his estimated valuation Ex_U,

Lemma 2. *Suppose the* informed *player plays the equilibrium strategy profile σ_I. If G is concave then $\sigma_U = \left(b_U^1, b_U^2\right)$ defines an equilibrium strategy profile in which the* uninformed *player overbids* *with respect to Ex_U at the first auction and bids truthfully at the second. Formally, $\sigma_U = \left(b_U^1, b_U^2\right)$ is defined by:*

$$\sigma_U = \begin{cases} b_U^1 \equiv b_0^* = Ex_U + \Gamma\left(b_0\right) \\ b_U^2 = \begin{cases} x_U & \text{if } b_0^* > b_I^1 \\ Ex_U & \text{otherwise} \end{cases} \end{cases}$$

with $\Gamma\left(b_0\right) = \left[1 - G\left(\phi\left(b_0\right)\right)\right]\left[E\left[X_U | X_U \geq \phi\left(b_0\right)\right] - \phi\left(b_0\right)\right] \geq 0$.

Proof. See appendix A.2. □

The opportunity to learn his valuation causes buyer U to bid aggressively during the first auction. It is optimal for him to overbid at the first auction with respect to his estimated valuation and to bid truthfully at the second auction. Bidder U accounts for the expected competitiveness of his *informed* competitor so that the overbid quantity, $|E\left[X_2\right] - b_0|$, equals the expected informational rent of an *informed* alter ego if he were to have a valuation greater than $\phi\left(b_0\right)$. Since X_is are *iid*, we can write $E\left[X_U | X_U \geq \phi\left(b_0\right)\right] - \phi\left(b_0\right) = E\left[X_I | X_I \geq \phi\left(b_0\right)\right] - \phi\left(b_0\right) = E\left[\beta_I^1\left(X_I\right) | \beta_I^1\left(X_I\right) \geq b_0\right] - b_0$. In other words, he bids a quantity that would correspond to his expected informational rent if he were to be *informed* and to compete against an *uninformed* buyer.

Equilibrium: The model results in an asymmetric equilibrium in which the *informed* buyer takes advantage of his knowledge and considers the potential benefits of the learning from his rival, whereas the latter conditioned his uninformative first-period bids on his expected informational rent if he were to be informed.

Proposition 1. *The strategy profile $\sigma = \left(\sigma_I, \sigma_U\right)$ forms an asymmetric Bayes-Nash equilibrium of a sequential second-price auction with asymmetrically informed buyers.*

$$\sigma_I = \begin{cases} b_I^1 < x_I \\ b_I^2 = x_I \end{cases}$$

$$\sigma_U = \begin{cases} b_U^1 = b_0^* \geq Ex_U \\ b_U^2 = \begin{cases} x_U & \text{if } b_0 > b_I^1 \\ Ex_U & \text{otherwise} \end{cases} \end{cases}$$

Proof. The result is based on lemmata 1 and 2. □

The first unit bears a value of consumption and an informational value for the *uniformed* player. As a result, along the equilibrium path, player U offers a price strictly higher than his expectations and beats with probability one all his opponent's types lower than Ex_U, while the *informed* player shades his first-round bid in regard to his private valuation. The aggressive behaviour of player U always entails information acquisition against all his opponent's types lower than Ex_U so that in equilibrium a low-*informed* type, i.e., $x_I < Ex_U$, always loses the first unit. In the last auction, both players act truthfully as player I bids his private valuation and player U either his realized information or his estimated valuation. Notice that since a low-*informed* type never wins the first unit, at least the mechanism maintains an *interim* efficient allocation.

3.2 Price trend

As pointed out at the beginning of the section, if players were to be symmetrically informed with constant valuations over units, the sequence of price (and revenues) should also remain constant. When players are asymmetrically informed, the equilibrium results in constant (*ex-ante*) expected equilibrium prices (on average they remain constant over both auctions). However, the next proposition shows that given the first price, the resulting price sequence is expected to decline.

Proposition 2. Ex-ante, *equilibrium prices are expected to be constant between both auctions, that is,* $E(p_1 - p_2) = 0$.

This proposition asserts that *prior* to the first auction game, both auctions' prices are, on average, the same so that the expected price path is constant. In equilibrium, the first auction can only be won by a high-type *informed* player, which implies a lower bid from player U in the second auction. To see why, note that if the first winner is a high-type player I, then $b_0^* \leq \beta_I^1(x_I) \Leftrightarrow \phi(b_0^*) \leq x_I$ and given that in equilibrium $b_0^* \geq Ex_U$ it follows that, by properties 1 and 2, $\phi(x_0) = Ex_U \leq \phi(b_0^*)$. Thus, if the first-period runner-up is player U, then in equilibrium $X_I \geq \phi(b_0^*) \geq \phi(x_0) = Ex_U$, which *de facto* implies that $P_2 = \min\{X_I, Ex_U\} = Ex_U < b_0^* = P_1$. Hence, the game results in buyer I being the winner of both units and the decline is driven by the opportunity to experience the unit.

However, if the *uninformed* buyer turns out to compete against a low-type player I, then he wins the first unit. Now, if $\beta_I^1(x_I) < b_0^*$, that is, player I becomes the runner-up, the second period equilibrium price is equal to $P_2 = \min\{X_I, X_U\}$. If in the second auction the *informed* buyer is the top performer then $Ex_U \geq X_I > X_U$ and depending on the likelihood that $\beta_I^1(x_I)$ is higher or lower than the realized value of X_U then we could have $P_1 \lesseqgtr P_2$. Conversely, if the low-type *informed* player is also the runner-up at the second auction, that is, $X_I < X_U$ then prices increase because in equilibrium $\beta_I^1(x_I) \leq x_I$.

Overall, each event offsets each other on average so that *ex-ante* the equilibrium prices are expected to remain constant between both sales. Note that along the equilibrium path, the proposition does not rule out possibilities for different patterns of price. If both units are won by the *informed* buyer prices are expected to be decreasing, whereas it is expected to be increasing if the *uninformed* buyer pockets both units. If units are won by different buyers, there is no clear tendency and both trends can happen in equilibrium.

Proposition 3. *Given the first auction price* p_1*, the expected second auction price is lower than the first auction price. The price sequence forms a* supermartingale, *that is* $E(p_2|p_1) \leq p_1$.

The underlying result of this proposition points out the possible existence of a declining-price sequence as the second auction price is expected to be lower than the realized first price p_1 (after the first auction game outcome is known). Consider that player I is the first round runner-up. If player I loses the first round, then $P_1 = \beta_I^1(x_I)$, thus we obtain,

$$
\begin{aligned}
E[P_2|p_1] &= E\left[\min\left\{\beta_I^2(X_I), \beta_U^2(X_U)\right\}|p_1 = \beta_I^1(X_I)\right] \\
&= E\left[X_I \mathbb{1}_{\{X_I < X_U\}}|\phi(p_1) = X_I\right] + E\left[X_U \mathbb{1}_{\{X_I > X_U\}}|\phi(p_1) = X_I\right],
\end{aligned}
$$

which gives,

$$E[P_2|p_1] = \phi(p_1)(1 - G(\phi(p_1))) + \int_0^{\phi(p_1)} x_U dG(x_U)$$
$$= x_I - \int_0^{x_I} G(x_U)\, dx_U.$$

Now, notice that $\forall x_I \in [0, Ex_U]$, relation (1) in appendix A.1 can be written as $\beta_I^1(x_I) = x_I - \int_0^{x_I} G(x_U)\, dx_U$. Together with property 1 we conclude that $\forall x_I \in [0, Ex_U]$ then $E[P_2|p_1] = p_1$ and that $\forall x_I \in [Ex_U, \bar{x}]$ then $E[P_2|p_1] < p_1$ since player I's equilibrium first period bid is strictly increasing. Suppose now that player U is the first auction runner-up, then $P_1 = b_0$ and $P_2 = \min\{X_I, Ex_U\}$. Since in equilibrium $b_0 > Ex_U$ we obtain that $p_1 = \beta_I^1(x_I) > Ex_U$, so that necessarily $X_I > Ex_U$, which implies that $E[P_2|p_1 = b_0] = Ex_U < p_1$. Hence, it follows that overall the contribution to the expected second price of each event implies that $E[P_2|p_1] \leq p_1$. The *uniformed* bidder's incentives to gather information are strong enough to make the first auction price relatively high, no matter how likely the equilibrium second auction price is in regard to the first price. Conditional on the first price, the expected second price is below the first one and prices follow a supermartingale.

4 Concluding remarks

The work analyses the bidding pattern of an *uninformed* buyer involved in a sequential second-price auction who may obtain exogenous information and proposes a tractable characterization of the resulting equilibrium bids. We propose an alternative and intuitive explanation for the observed decline in price at auction that has not been proposed in the literature for a private values second-price rule. It is shown that (*i*) the unaware buyer places higher bids in the first auction than if he were perfectly *informed* whereas his rival adopts a more conservative behaviour by using bid shading for different motivations than if he were to compete against an *informed* competitor, and (*ii*) that equilibrium prices should follow a decreasing path with positive probability driven by the opportunity to learn.

The shading under standard arguments comes from the indifference between losing and winning the first stage provided his uninformed competitor has the same valuation. As a result, the *informed* buyer bids what he expects to pay during the second auction should he have the highest valuation. Conversely, preventing the learning of the *uninformed* buyer might be detrimental for low-*informed* types. Then, the *informed* buyer trades-off the cost of winning and earns a strict positive payoff in the second auction with positive probability by losing the first unit. It is optimal for him to lose the first unit and to let his opponent acquires information by reducing his first-period bid. While it increases the probability of being runner-up of the first auction game, it increases the probability of winning the second unit at a cheaper price. Conversely, the *uninformed* buyer acts myopically and cares only about the realized information he could earn by winning the first stage. As a result, he sets an optimal price offer strictly higher than his estimated valuation, driven by the opportunity to gather positive information in the future. The *uninformed* buyer gets carried away and such aggressiveness is at the root of the declining equilibrium price sequence and allocative inefficiency.

Finally, from an allocative perspective, the proposed equilibrium does not convey *ex-post* allocative efficiency. Naturally, an *uninformed* buyer that loses might have turned out to be the winning buyer if he had learned his valuation. Also, the proposed equilibrium does not constitute an *ex-post* equilibrium as the revealed information might end up being bad news for the *uninformed* buyer so that he might regret offering a quantity strictly higher than his expected valuation.

This work rests on the main assumption of a costless and exogenous information acquisition in the case of a win. The idea of acquiring perfect knowledge once this buyer wins a unit was created on the basis that objects can be thought of as experience goods from which perfect information is gathered up only by using it. The costless acquisition is then driven on the basis of full rationality and full capability of a buyer in processing the information gleaned when using the good. A way of improvement would allow either for partial information discovery or would introduce an exogenous cost affecting to the discovery. Influencing the value of discovery or its amount also entails the issue of an active strategic seller which could manage to affect the *ex-ante* private learning, and private learning in the course of the auction by disclosing information. No work has yet investigated the issue of the optimal value discovery and learning under the framework of sequential private values second-price auctions with multi-unit demand buyers.

A Appendix

A.1 Proof of lemma 1

We look for a Bayesian equilibrium in which player I with valuation x_I chooses a bid $b_I^1 = \beta_I^1(x_I)$ to maximize his expected payoff. From his viewpoint buyer U acts as a dummy player who submits a quantity b_0 drawn from a *cdf* $F(.) \in [0, \bar{x}]$. Assume that for the first round he chooses to act as type $z \geq x_I$ and bids $b = \beta_I^1(z)$. His overall expected payoff becomes:

$$\begin{aligned}\Pi_I^+(z; x_I) = &\int_0^{\beta_I^1(z)} \left(x_I - b_0 + (x_I - Ex_U)^+\right) dF(b_0) \\ &+ \left(1 - F\left(\beta_I^1(z)\right)\right) \int_0^{x_I} (x_I - x_U)\, dG(x_U)\end{aligned}$$

The first integral corresponds to the event in which he wins both periods against $b_U^1 = b_0$ in the first round and $b_U^2 = Ex_U$ in the second one. The second one corresponds to the event in which he has lost the first round against $b_U^1 = b_0$ and wins the second auction against $\beta_U^2(x_U) = x_U$ by bidding truthfully $\beta_I^2(x_I) = x_I$.

If he acts as type $z < x_I$ and bids $b = \beta_I^1(z)$ then his payoff becomes:

$$\begin{aligned}\Pi_I^-(z; x_I) = &\int_0^{\beta_I^1(z)} \left(x_I - b_0 + (x_I - Ex_U)^+\right) dF(b_0) \\ &+ (1 - F(x_I)) \int_0^{x_I} (x_I - x_U)\, dG(x_U) \\ &+ \int_{\beta_I^1(z)}^{x_I} \int_0^{x_I} (x_I - x_U)\, dG(x_U)\, dF(t)\end{aligned}$$

The first integral is identical to the previous case. The second line expresses the situation in which he loses the first round against a bid $b_0 > x_I > b$ and the third line denotes the situation in which buyer I loses the first round against a bid $b < b_0 \leq x_I$.

If for the *informed* player it is optimal to act as type $z = x_I$ then it should be the case that:

$$\left.\frac{\partial \Pi_I^+ (z; x_I)}{\partial z}\right|_{z=x_I} \leq 0 \quad and \quad \left.\frac{\partial \Pi_I^- (z; x_I)}{\partial z}\right|_{z=x_I} \geq 0$$

both FOCs are given by:

$$\left.\frac{\partial \Pi_I^+ (z; x_I)}{\partial z}\right|_{z=x_I} : \beta_I^{1'} (x_I) f \left(\beta_I^1 (x_I)\right) \left(\left(x_I - \beta_I^1 (x_I) + (x_I - Ex_U)^+\right) - \int_0^{x_I} (x_I - x_U) \, dG (x_U)\right) \leq 0$$
$$\left.\frac{\partial \Pi_I^- (z; x_I)}{\partial z}\right|_{z=x_I} : \beta_I^{1'} (x_I) f \left(\beta_I^1 (x_I)\right) \left(\left(x_I - \beta_I^1 (x_I) + (x_I - Ex_U)^+\right) - \int_0^{x_I} (x_I - x_U) \, dG (x_U)\right) \geq 0$$

which lead to the following equilibrium strategy at $z = x_I$:

$$\begin{aligned} \beta_I^1 (x_I) &= x_I (1 - G (x_I)) + (x_I - Ex_U)^+ + E [X_U | X_U \leq x_I] \, G (x_I) \\ &= x_I + (x_I - Ex_U)^+ - \int_0^{x_I} G (x_U) \, dx_U \end{aligned} \tag{1}$$

To ensure that the proposed bid function constitutes an equilibrium strategy candidate, substitute the expression of $\beta_I^1 (x_I)$ at z in the *FOC*. Doing so we obtain the following:

$$\begin{aligned} \frac{\frac{\partial \Pi_I (z; x_I)}{\partial z}}{\beta_I^{1'} (z) f \left(\beta_I^{1'} (z)\right)} = \Psi (z) &= x_I - \beta_I^1 (z) + (x_I + Ex_U)^+ - \int_0^{x_I} (x_I - x_U) \, dG (x_U) \\ &= x_I - z - (z - Ex_U)^+ + (x_I - Ex_U)^+ - \int_0^{x_I} dG (x_U) + \int_0^{z} dG (x_U) \end{aligned}$$

Suppose that $z > x_I > Ex_U$ then we obtain:

$$\begin{aligned} \Psi (z) &= x_I - \beta_I^1 (z) + (x_I + Ex_U)^+ - \int_0^{x_I} (x_I - x_U) \, dG (x_U) \\ &= 2 (x_I - z) + \int_{x_I}^{z} dG (x_U) \leq 0 \end{aligned}$$

If $Ex_U > z > x_I$ then:

$$\Psi (z) = x_I - z + \int_{x_I}^{z} dG (x_U) = (z - x_I) (G (z) - 1) \leq 0$$

It turns out that for any type $z > x_I$, player I can increase his payoff by acting as his true type. It is not optimal to mimic any higher type. Consider now the situation $Ex_U < z < x$, then we also obtain:

$$\Psi (z) = 2 (x_I - z) - \int_{z}^{x_I} dG (x_U) \geq 0$$

If $z < x_I < Ex_U$ then:

$$\Psi (z) = (x_I - z) - \int_{z}^{x_I} dG (x_U) \geq 0$$

Finally, if the following situation $z < Ex_U < x_I$ is realized, then we obtain the following:

$$\Psi (z) = (x_I - z) + (x_i - Ex_U)^+ - \int_{z}^{x_I} dG (x_U) \geq 0$$

As a result, mimicking any type z lower than his true type is also not optimal. He can also increase his payoff by acting as his true type. This shows that $\sigma_I^* = \left(b_I^1, b_I^2\right)$ forms an equilibrium strategy profile.

To see now that $\beta_I^1(x_I) \leq x_I$ notice that if $x_I < Ex_U$ we have that $\beta_I^1(x_I) = x_I - \int_0^{x_I} G(x_U)\, dx_U \leq x_I$. Then, if $x_I \geq Ex_U$ we have that

$$\begin{aligned}\beta_I^1(x_I) &= 2x_I - Ex_U - \int_0^{x_I} G(x_U)\, dx_U \\ &= 2x_I - \bar{x} + \int_{x_I}^{\bar{x}} G(x_U)\, dx_U \leq 2x_I - \bar{x} + (\bar{x} - x_I) = x_I\end{aligned}$$

and the assertion that the *informed* player underbids at the first auction follows.

A.2 Proof of lemma 2

The proof consists of three parts. We first prove that for some range of first-period bids $b_U^1 = b_0$ the *uninformed* player's expected utility payoff function is strictly increasing so that there is no best-response for this range. We next prove that his expected utility payoff is a concave function for the opposite range of bid values and that a unique maximand exists within this set. Finally, we show that the optimal first-period bid b_0^* involves an equilibrium quantity greater than Ex_U. As a matter of technical simplification, consider the following property:

Property 2. $\exists x_0 \in [0, \bar{x}] : \forall b_0 \geq x_0$ *and* $b_0 \leq \beta_I^1(x_I)$ *,* $\phi(b_0) \geq Ex_U \Rightarrow X_I \in [Ex_U, \bar{x}]$

Proof. Denote by $\beta_I^-(x_I)$ and $\beta_I^+(x_I)$ respectively player I's bid function $\forall x_I \in [0, Ex_U]$ and $\forall x_I \in [Ex_U, 0]$. We have for $\tilde{x} = Ex_U$ that $\beta_I^-(\tilde{x}) = \beta_I^+(\tilde{x}) = x_0 \Leftrightarrow \gamma(x_0) = \phi(x_0) = \tilde{x} = Ex_U$. By property 1, $\phi(b_0) > \phi(x_0) = Ex_U > x_0$, hence $\beta_I^1(x_I) \geq b_0 \geq x_0 \Leftrightarrow x_I \geq \phi(b_0) \geq \phi(x_0) = Ex_U$. The explicit value of x_0 is simply equals to: $\phi(x_0) = Ex_U \Leftrightarrow x_0 = \beta_I^1(\tilde{x}) \Leftrightarrow x_0 = \tilde{x} + (\tilde{x} - Ex_U)^+ - \int_0^{\tilde{x}} (\tilde{x} - x_U)\, dG(x_U) = \tilde{x} - \int_0^{\tilde{x}} G(x_U)\, dx_U$, in which the last equality follows from integration by parts. $\square$

This property says that there exists some threshold $x_0 > Ex_U$ so that any first period losing bid b_0 higher than x_0 implies that player I's valuation is at least equal to the expected value of X_U. Therefore, any truthful bid $b_U^2 = Ex_U$ in the second period after losing the first unit is a losing bid. Under property 2 we can split buyer U 's decision easily. It is considered the expected utility payoff function $\Pi_U^-(b_0)$ from playing any quantity $b_0 \leq x_0$ and the expected utility payoff function $\Pi_U^+(b_0)$ from playing any quantity $b_0 > x_0$.

Any bid b_0 in the range $[0, x_0]$ leads to the following expected utility payoff function:

$$\begin{aligned}\Pi_U^-(b_0) = &\int_0^{\gamma(b_0)} \int_{x_I}^{\bar{x}} \left(2x_U - \beta_I^1(x_I) - x_I\right) dG(x_U)\, dG(x_I) \\ &+ \int_0^{\gamma(b_0)} \int_0^{x_I} \left(x_U - \beta_I^1(x_I)\right) dG(x_U)\, dG(x_I) \\ &+ \int_{\gamma(b_0)}^{Ex_U} \int_{\gamma(b_0)}^{Ex_U} \int_0^{\bar{x}} (x_U - x_I)\, dG(x_U)\, dG(x_I)\, dG(t) \\ &+ \int_{Ex_U}^{\bar{x}} (0)\, dG(t)\end{aligned}$$

Considering the first two lines, buyer U wins both units if in the second auction $x_U > X_I$ and only the first one otherwise. The third one addresses the case in which he loses the first unit but still wins the second with a bid Ex_U, which is the conjunction of two events: buyer U can lose the first auction if

$$\begin{array}{ll} (i) & Ex_U > x_0 > \beta_I^1(x_I) > b_0 \\ (ii) & Ex_U > \beta_I^1(x_I) > x_0 > b_0 \end{array}$$

The situation in which $\beta_I^1(x_I) > Ex_U > x_0 > b_0$ is realized implies a certain loss in the second period.

(*i*) No best-response in the range of $\Pi_U^-(b_0)$. We prove first that there is no best-response for the *uninformed* bidder within the range $[0, x_0]$ by showing that $\Pi_U^-(b_0)$ is strictly increasing in b_0. Differentiate Π_U^- over b_0 we obtain:

$$\frac{\partial \Pi_U^-}{\partial b_0} = g(\gamma(b_0)) \frac{\partial \gamma(b_0)}{\partial b_0} \left[\int_{\gamma(b_0)}^{\bar{x}} (2x_U - b_0 - \gamma(b_0))\, dG(x_U) + \int_0^{\gamma(b_0)} (x_U - b_0)\, dG(x_U) \right.$$
$$\left. - \int_{\gamma(b_0)}^{Ex_U} (Ex_U - x_I)\, dG(x_I) + \gamma(b_0) \int_{\gamma(b_0)}^{Ex_U} g(t)\, dt - Ex_U \int_{\gamma(b_0)}^{Ex_U} g(t)\, dt \right]$$

The product $g(\gamma(b_0)) \frac{\partial \gamma(b_0)}{\partial b_0}$ is strictly positive, and rearranging the term in brackets and replacing $Ex_U = \gamma(x_0)$ we get relation $T(b_0) = \gamma(x_0) - b_0 + A(b_0) - B(b_0)$, in which

$$\begin{aligned} A(b_0) &= \int_{\gamma(b_0)}^{\bar{x}} (x - \gamma(b_0))\, dG(x) \\ B(b_0) &= \int_{\gamma(b_0)}^{\gamma(x_0)} (2\gamma(x_0) - x - \gamma(b_0))\, dG(x) \end{aligned}$$

The problem is nested to show that this relation is positive, thus if it is positive at the lower boundary and at the upper one, it suffices to show that the relation is monotonic so that it is positive everywhere. Let us first compute the value of this function at both $b_0 = 0$ and $b_0 = x_0$. For $b_0 = 0$ we obtain:

$$T(0) = Ex_U + \int_0^{\bar{x}} x dG(x) - \int_0^{Ex_U} (2Ex_U - x)\, dG(x)$$

since $\gamma(0) = 0$. Rearranging the relation we obtain:

$$T(0) = 2Ex_U - 2Ex_U \int_0^{Ex_U} dG(x) + \int_0^{Ex_U} x dG(x) > 0$$

Now for $b_0 = x_0$, that is, $\gamma(x_0) = Ex_U$, we obtain:

$$T(x_0) = Ex_U - x_0 + \int_{Ex_U}^{\bar{x}} (x - Ex_U)\, dG(x) > 0$$

So the relation is positive at its two boundaries. Next, we show that $T(b_0)$ is monotonic between these boundaries. By integration by parts, $T(b_0)$ can be rearranged as:

$$\begin{aligned}
T(b_0) &= Ex_U - b_0 + \int_{\gamma(b_0)}^{\bar{x}} (x - \gamma(b_0))\, dG(x) - \int_{\gamma(b_0)}^{Ex_U} (2Ex_U - x - \gamma(b_0))\, dG(x) \\
&= Ex_U - b_0 + [\bar{x} - \gamma(b_0) G(\gamma(b_0))] - \int_{\gamma(b_0)}^{\bar{x}} G(x)\, dx \\
&- \gamma(b_0) \int_{Ex_U}^{\bar{x}} dG(x) - 2Ex_U \int_{\gamma(b_0)}^{\bar{x}} dG(x) + \int_{\gamma(b_0)}^{Ex_U} x dG(x) \\
T(b_0) &= Ex_U - b_0 + [\bar{x} - \gamma(b_0) G(\gamma(b_0))] - \int_{\gamma(b_0)}^{\bar{x}} G(x)\, dx \\
&- \gamma(b_0)[1 - G(Ex_U)] - 2Ex_U [G(Ex_U) - G(\gamma(b_0))] \\
&+ [Ex_U G(Ex_U) - \gamma(b_0) G(\gamma(b_0))] - \int_{\gamma(b_0)}^{Ex_U} G(x)\, dx
\end{aligned}$$

We now differentiate the relation $T(b_0)$. First, notice that $G'(\gamma(b_0)) = \gamma'(b_0) g(\gamma(b_0))$, we thus obtain:

$$\begin{aligned}
T'(b_0) &= -1 - \gamma'(b_0) G(\gamma(b_0)) - \gamma(b_0) g(\gamma(b_0)) \gamma'(b_0) + \gamma'(b_0) G(\gamma(b_0)) \\
&- \gamma'(b_0)[1 - G(Ex_U)] + 2Ex_U \gamma'(b_0) g(\gamma(b_0))[1 - G(Ex_U)] \\
&- \gamma'(b_0) G(\gamma(b_0)) - \gamma(b_0) g(\gamma(b_0)) \gamma'(b_0) + \gamma'(b_0) G(\gamma(b_0))
\end{aligned}$$

giving

$$\begin{aligned}
T'(b_0) &= \gamma'(b_0)[-G(\gamma(b_0)) + G(\gamma(b_0)) - \gamma(b_0) g(\gamma(b_0)) - [1 - G(Ex_U)] \\
&\quad +2Ex_U g(\gamma(b_0)) - G(\gamma(b_0)) + G(\gamma(b_0)) - \gamma(b_0) g(\gamma(b_0))] - 1 \\
&= \gamma'(b_0)[2g(\gamma(b_0))[Ex_U - \gamma(b_0)] - [1 - G(Ex_U)]] - 1
\end{aligned}$$

Hence, if $T'(b_0) < 0$ we are done. Now, maximizing/minimizing the relation:

$$H(b_0) = [2g(\gamma(b_0))[Ex_U - \gamma(b_0)] - [1 - G(Ex_U)]]$$

we obtain the following:

$$\begin{aligned}
H'(b_0):\ Ex_U - \gamma(b_0) - \frac{g(\gamma(b_0))}{g'(\gamma(b_0))} &= 0 \\
\gamma(b_0)^* &= Ex_U - \frac{g(\gamma(b_0))}{g'(\gamma(b_0))}
\end{aligned}$$

Notice that $\gamma(b_0)^* > 0$ if $g'(\gamma(b_0)) < 0$, which is satisfied if the function G is concave. Also, notice that if so then $H'(b_0) > 0$ since $Ex_U > \gamma(b_0)$ and $-\frac{g(\gamma(b_0))}{g'(\gamma(b_0))} > 0$, which implies that we can obtain $\gamma(b_0)^*$ as a minimand if $H(b_0)$ is a convex function, that is, $H''(b_0) > 0$. We have that:

$$\begin{aligned}
H''(b_0) &= -\gamma'(b_0) + \frac{g'(\gamma(b_0))\gamma'(b_0) g'(\gamma(b_0)) - g(\gamma(b_0))\gamma'(b_0) g''(\gamma(b_0))}{[g'(\gamma(b_0))]^2} \\
&= \gamma'(b_0) \frac{g(\gamma(b_0)) g''(\gamma(b_0))}{[g'(\gamma(b_0))]^2} - 2\gamma'(b_0)
\end{aligned}$$

hence, $H''(b_0) < 0$ if $g''(\gamma(b_0)) < 0$ which is not possible otherwise g cannot be a density function of a concave *cdf* since $g'(\gamma(b_0)) < 0$. Thus, $H''(b_0) > 0$ if the following is satisfied:

$$g''(\gamma(b_0)) \geq \frac{2\left[g'(\gamma(b_0))\right]^2}{g(\gamma(b_0))}$$

if so we have $\gamma(b_0)^*$ as a minimand. Plug $\gamma(b_0)^*$ in $H(b_0)$ and replace it in $T(b_0)$, we obtain the following:

$$\begin{aligned} T'(b_0) &= \gamma'(b_0)\left[2g(\gamma(b_0))\left[Ex_U - \gamma(b_0)^*\right] - \left[1 - G(Ex_U)\right]\right] - 1 \\ &= \gamma'(b_0)\left[\frac{2\left[g(\gamma(b_0))\right]^2}{g'(\gamma(b_0))} - \left[1 - G(Ex_U)\right]\right] - 1 \end{aligned}$$

Hence, we have that $\gamma'(b_0) > 0$, so in order for $T'(b_0)$ to be negative we need $g'(\gamma(b_0)) < 0$, that is, G to be a concave function. Moreover, $\gamma(b_0)^*$ is a minimand which ensure us that we have maximized the above difference. Thus, $T(b_0)$ is monotonically decreasing over the domain. Therefore, $\Pi^-(b_0)$ is is monotonically increasing for all $b_0 \in [0, x_0]$.

(*ii*) Existence of a maximand in the range of $\Pi_U^+(b_0)$. Consider now all bids $b_0 \geq x_0$. Then, according to property 2 any losing bid in the first auction implies a strict loss in the second one. This leads to the following overall expected payoff:

$$\begin{aligned} \Pi_U^+(b_0) = &\int_{\phi(x_0)}^{\phi(b_0)} \int_{x_I}^{\bar{x}} \left(2x_U - \beta_I^1(x_I) - x_I\right) dG(x_U)\, dG(x_I) \\ &+ \int_{\phi(x_0)}^{\phi(b_0)} \int_{0}^{x_I} \left(x_U - \beta_I^1(x_I)\right) dG(x_U)\, dG(x_I) \\ &+ \int_{0}^{\phi(x_0)} \int_{x_I}^{\bar{x}} \left(2x_U - \beta_I^1(x_I) - x_I\right) dG(x_U)\, dG(x_I) \\ &+ \int_{0}^{\phi(x_0)} \int_{0}^{x_I} \left(x_U - \beta_I^1(x_I)\right) dG(x_U)\, dG(x_I) \\ &+ \int_{\phi(b_0)}^{\bar{x}} (0)\, dG(x_I) \end{aligned}$$

Differentiating $\Pi_U^+(b_0)$ over b_0, we obtain the following *FOC*:

$$b_0 - \int_0^{\phi(b_0)} x_U g(x_U)\, dx_U - 2\int_{\phi(b_0)}^{\bar{x}} x_U g(x_U)\, dx_U + \phi(b_0)\left[1 - G(\phi(b_0))\right] = 0$$

Rearranging the formulation, we obtain the relation of the proposition:

$$b_0^* = Ex_U + \Gamma(b_0) \tag{2}$$

with $\Gamma(b_0) = \left[1 - G(\phi(b_0))\right]\left[E\left[X_U | X_U \geq \phi(b_0)\right] - \phi(b_0)\right]$.

(*ii*) Overbidding is optimal. It is now straightforward to notice that in relation (2), $\Gamma(b_0) \geq 0$. Therefore, $b_0^* \geq Ex_U$ and the assertion that it is optimal for the *uninformed* player to overbid with respect to his expected valuation at the first auction stage follows.

References

Ashenfelter, O.: 1989, How auctions work for wine and art, *The journal of Economic Perspectives* **3**(3), 23–26.

Beggs, A. and Graddy, K.: 1997, Declining values and the afternoon effect: Evidence from art auctions, *The RAND Journal of Economics* **28**(3), 544–565.

Bernhardt, D. and Scoones, D.: 1994, A note on sequential auctions, *The American Economic Review* **84**(3), 653–657.

Bikhchandani, S.: 1988, Reputation in second-price auctions, *Journal of Economic Theory* **46**(1), 97–119.

Black, J. and de Meza, D.: 1992, Systematic price differences between successive auctions are not anomaly, *Journal of Economics & Management Strategy* **1**(4), 607–628.

Celis, L. E., Lewis, G., Mobius, M. M. and Nazerzadeh, H.: 2011, Buy-it-now or take-a-chance: a simple sequential screening mechanism, *Proceedings of the 20th international conference on World wide web*, ACM, pp. 147–156.

Deltas, G. and Kosmopoulou, G.: 2004, "catalogue" vs "order-of-sale" effects in sequential auctions: theory and evidence from a rare book sale, *The Economic Journal* **114**(492), 28–54.

Ding, W., Jeitschko, T. D. and Wolfstetter, E. G.: 2010, Signal jamming in a sequential auction, *Economics Letters* **108**(1), 58–61.

Engelbrecht-Wiggans, R.: 1994, Sequential auctions of stochastically equivalent objects, *Economics Letters* **44**(1-2), 87–90.

Engelbrecht-Wiggans, R. and Weber, R. J.: 1983, A sequential auction involving asymmetrically-informed bidders, *International Journal of Game Theory* **12**(2), 123–127.

Février, P.: 2003, He who must not be named, *Review of Economic Design* **8**(1), 99–119.

Gandal, N.: 1997, Sequential auctions of interdependent objects: Israeli cable television licenses, *The Journal of Industrial Economics* **45**(3), 227–244.

Hausch, D. B.: 1986, Multi-object auctions: Sequential vs. simultaneous sales, *Management Science* **32**(12), 1599–1610.

Hörner, J. and Jamison, J.: 2008, Sequential common-value auctions with asymmetrically informed bidders, *The Review of Economic Studies* **75**(2), 475–498.

Jeitschko, T. D.: 1998, Learning in sequential auctions, *Southern Economic Journal* **65**(1), 98–112.

Jeitschko, T. D.: 1999, Equilibrium price paths in sequential auctions with stochastic supply, *Economics Letters* **64**(1), 67–72.

Jeitschko, T. D. and Wolfstetter, E. G.: 2002, Scale economies and the dynamics of recurring auctions, *Economic Inquiry* **40**(3), 403–414.

Kannan, K. N.: 2010, Declining prices in sequential auctions with complete revelation of bids, *Economics Letters* **108**(1), 49–51.

Kittsteiner, T., Nikutta, J. and Winter, E.: 2004, Declining valuations in sequential auctions, *International Journal of Game Theory* **33**(1), 89–106.

Lewis, R. A. and Rao, J. M.: 2015, The unfavorable economics of measuring the returns to advertising, *The Quarterly Journal of Economics* **130**(4), 1941–1973.

McAfee, R. P.: 2011, The design of advertising exchanges, *Review of Industrial Organization* **39**(3), 169–185.

McAfee, R. P. and Vincent, D.: 1993, The declining price anomaly, *Journal of Economic Theory* **60**, 191–212.

Menezes, F. M. and Monteiro, P. K.: 2003, Synergies and price trends in sequential auctions, *Review of Economic Design* **8**(1), 85–98.

Milgrom, P. R. and Weber, R. J.: 2000, A theory of auctions and competitive bidding ii. mimeo.

Prat, A. and Valletti, T. M.: 2018, Attention oligopoly, *Available at ssrn: https://ssrn.com/abstract=3197930*.

Raviv, Y.: 2006, New evidence on price anomalies in sequential auctions: Used cars in new jersey, *Journal of Business & Economic Statistics* **24**(3), 301–312.

von der Fehr, N.-H. M.: 1994, Predatory bidding in sequential auctions, *Oxford Economic Papers* **46**(3), 345–356.

Weber, R. J.: 1983, Multiple-object auctions, *in* R. Engelbrecht-Wiggans, M. Shubik and R. M. Stark (eds), *Auctions, Bidding and Contracting: Uses and Theory*, New York University Press, pp. 165–191.

Chaire Gouvernance et Régulation
Fondation Paris-Dauphine
Place du Maréchal de Lattre de Tassigny - 75016 Paris (France)
http://chairgovreg.fondation-dauphine.fr